

## MATHEMATTIC SOLUTIONS

*Statements of the problems in this section originally appear in 2021: 47(2), p. 72–73.*

**MA107.** A wooden cube is painted red on five of its six sides and then cut into identical small cubes, of which 52 have exactly two red sides. How many small cubes have no red sides?

*Originally Question 27 of 2013 University of Cape Town Mathematics Competition (Grade 12).*

*We received 9 submissions of which 6 were correct and complete. We present the solution by the Missouri State University Problem Solving Group.*

Denote the number of small cubes along an edge of the original cube by  $n$  (so there are a total of  $n^3$  small cubes). For concreteness, assume that the unpainted face of the original cube is the top face. A small cube has exactly two red sides if it is on one of the four vertical edges, but is not one of the four corner cubes on the bottom face (there are  $4(n-1)$  of these) or if it is on one of the edges of the bottom face, but is not one of the four corner cubes (there are  $4(n-2)$  of these). Thus  $4(n-1) + 4(n-2) = 52$  and hence  $n = 8$ . A small cube has no paint on it if it is in the interior of the cube (there are  $6^3 = 216$  of these) or it is in the interior of the top (unpainted) face (there are  $6^2 = 36$  of these). This gives a total of 252 unpainted small cubes.

For the record, there are four cubes with three faces painted and 204 cubes with one face painted.